Exercise 39

Find the critical numbers of the function.

$$F(x) = x^{4/5}(x-4)^2$$

Solution

A critical number is a value of x for which the derivative is zero or nonexistent. Take the derivative of the function.

$$\begin{aligned}
F'(x) &= \frac{d}{dx}[x^{4/5}(x-4)^2] \\
&= \left[\frac{d}{dx}(x^{4/5})\right](x-4)^2 + x^{4/5}\left[\frac{d}{dx}(x-4)^2\right] \\
&= \left(\frac{4}{5}x^{-1/5}\right)(x-4)^2 + x^{4/5}\left[2(x-4)^1 \cdot \frac{d}{dx}(x-4)\right] \\
&= \frac{4(x-4)^2}{5x^{1/5}} + x^{4/5}[2(x-4)\cdot(1)] \\
&= \frac{4(x-4)^2}{5x^{1/5}} + 2(x-4)x^{4/5} \times \frac{5x^{1/5}}{5x^{1/5}} \\
&= \frac{4(x-4)^2}{5x^{1/5}} + \frac{10(x-4)x}{5x^{1/5}} \\
&= \frac{4(x-4)^2 + 10(x-4)x}{5x^{1/5}} \\
&= \frac{4(x^2-8x+16) + (10x^2-40x)}{5x^{1/5}} \\
&= \frac{(4x^2-32x+64) + (10x^2-40x)}{5x^{1/5}} \\
&= \frac{14x^2-72x+64}{5x^{1/5}}
\end{aligned}$$

Set what's in the numerator and denominator equal to zero and solve for x.

$$\begin{aligned}
14x^2 - 72x + 64 &= 0 & \qquad 5x^{1/5} &= 0 \\
2(7x^2 - 36x + 32) &= 0 & x &= 0 \\
2(7x-8)(x-4) &= 0 & x &= 0 \\
x = \frac{8}{7} \quad \text{or} \quad x &= 4 & x &= 0
\end{aligned}$$